MATHEMATICS

1_. Nearest point will be foot of perpendicular drawn from centre.

$\frac{x-0}{2} = \frac{y-6}{1} = \frac{-(0+6+4)}{4+1} \Rightarrow (x, y) = (-4, 4)$.

2_. Let coordinates of centre is (h, k)

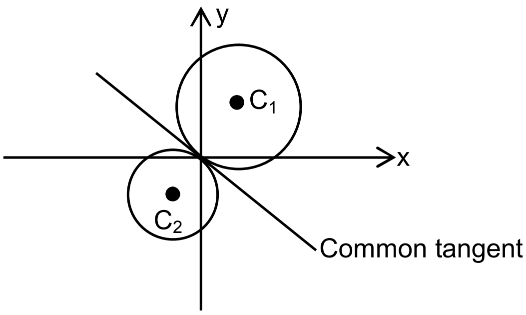
$\left|\frac{5h+12k-10}{13}\right| = 3$ and $\left|\frac{5h-12k-40}{13}\right| = 3$

$\Rightarrow$ 5h + 12k – 10 = 39 & –(5h – 12k – 40) = 39 $\Rightarrow$ (h, k) = (5, 2)

3_. Point C(a, a + 1) must lie outside the circle $x^2 + y^2 = 4$.

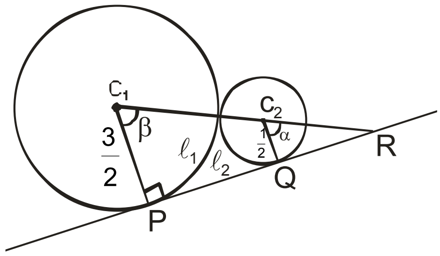
$\Rightarrow$ $S_1 > 0$ $\Rightarrow$ $a^2 + (a+1)^2 - 4 > 0$ $\Rightarrow$ $a < \frac{-1-\sqrt{7}}{2}$ or $a > \frac{-1+\sqrt{7}}{2}$

4_.



$\left.\frac{dy}{dx}\right|_{(0,0)} = -\frac{(3+\sin\beta)}{2\cos\alpha} = -\frac{2\cos\alpha}{2c} \Rightarrow c = \frac{2\cos^2\alpha}{3+\sin\beta}$ $\therefore$ $c_{max} = 1$.

5_.



$PQ = \sqrt{C_1C_2^2 - (r_1 - r_2)^2} = \sqrt{4-1} = \sqrt{3}$

$\frac{RC_1}{RC_2} = \frac{3/2}{1/2}$ $\Rightarrow$ $RC_1 = 3RC_2$ $\Rightarrow$ $RC_2 = 1$

$\therefore$ $\cos\alpha = \frac{1/2}{1}$ $\Rightarrow$ $\alpha = \frac{\pi}{3} = \beta$

Required perimeter $= \ell_1 + \ell_2 + \sqrt{3} = \frac{3}{2}.\frac{\pi}{3} + \frac{1}{2}.\frac{2\pi}{3} + \sqrt{3} = \frac{5\pi}{6} + \sqrt{3}$

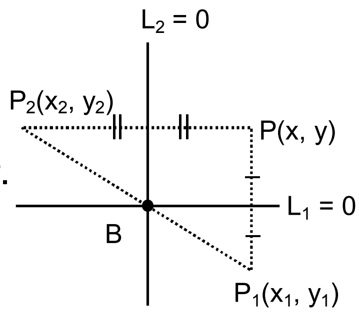
6. OA = OB = OC $\Rightarrow$ (0, 0) is circumcentre

as OG : GH = 1 : 2 $\Rightarrow$ H = 3G

$\Rightarrow$ $x = 3 + 5\cos\theta + 5\sin\theta$ and $y = 4 + 5\sin\theta - 5\cos\theta$

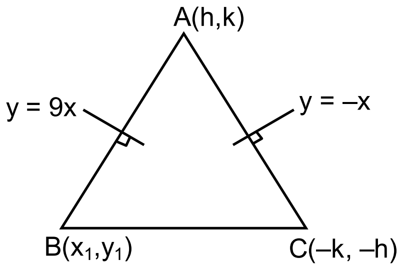
$\Rightarrow$ $x + y = 7 + 10\sin\theta$ and $x - y = -1 + 10\cos\theta$ $\Rightarrow$ $(x + y - 7)^2 + (x - y + 1)^2 = 10^2$

7.



$\because$ B is circumcentre of triangle PP_1P_2

8.



$x_1 = \dfrac{9k - 40h}{41}$, $y_1 = \dfrac{9h + 40k}{41}$

BC का समीकरण

$y + h = \dfrac{50h + 40k}{50k - 40h}(x + k)$

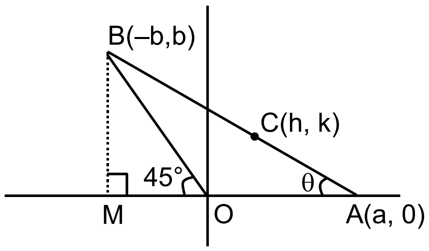
(f, g) lies on BC $\Rightarrow$ $g + h = \dfrac{5h + 4k}{5k - 4h}(f + k)$

$\Rightarrow$ locus of (h, k) is $4x^2 + 4y^2 + x(5f + 4g) + y(4f - 5g) = 0$ है ।

9. $a = (\sqrt{b} + \sqrt{c})^2 \Rightarrow$ $(\sqrt{a} - \sqrt{b} - \sqrt{c})(\sqrt{a} + \sqrt{b} + \sqrt{c}) = 0$ $\Rightarrow$ $\sqrt{a} - \sqrt{b} - \sqrt{c} = 0$

$\Rightarrow$ $(\sqrt{b} + \sqrt{c})x + \sqrt{b}\ \ y + \sqrt{c} = 0$ $\Rightarrow$ $\sqrt{b}(x + y) + \sqrt{c}(x + 1) = 0$

10.



$BM = 2\sin\theta$ $\Rightarrow$ $MO = 2\sin\theta$

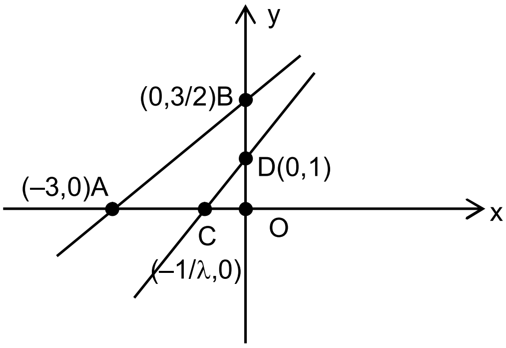
$MA = 2\cos\theta$

$\therefore$ $A(2\cos\theta - 2\sin\theta, 0)$ तथा $B(-2\sin\theta, 2\sin\theta)$

$\therefore$ $2h = 2\cos\theta - 4\sin\theta$ तथा $2k = 2\sin\theta$

As $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow$ $k^2 + (h + 2k)^2 = 1$ $\Rightarrow$ $h^2 + 5k^2 + 4hk = 1$

11. P lies on circle $x^2 + y^2 = c^2$. As curve is symmetrical about y = x and y = –x. So locus of Q and R will remain same.

12_*. 

Case – I If $\frac{-1}{\lambda} \neq -3$ i.e., $\lambda \neq \frac{1}{3}$, then

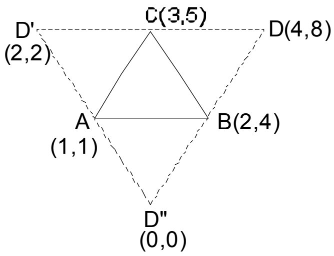
OA · OC = OB · OD $\Rightarrow$ $\frac{3}{\lambda} = \frac{3}{2}$ $\Rightarrow$ $\lambda = 2$.

Case-II If $\lambda = \frac{1}{3}$, then a unique circle will always pass through these point.

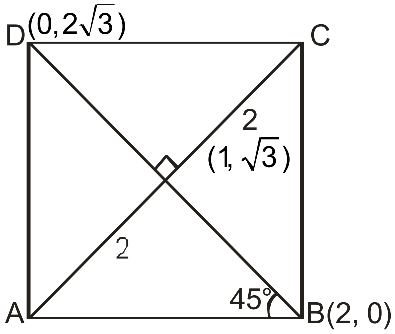
13_. $4a^2 - 2(3c - 2b)a + (b^2 + 2c^2 - 3bc) = 0$

$\Rightarrow$ $a = \frac{2(3c-2b) \pm \sqrt{4(3c-2b)^2 - 4.4.(b^2+2c^2-3bc)}}{2.4}$

$\Rightarrow$ $2a + b - 2c = 0$ & $2a + b - c = 0$

14_. 

15_. $\begin{vmatrix} t^2 & t & 6 \\ 2 & 3 & 5 \\ 3 & -2 & 1 \end{vmatrix} = 0 \Rightarrow$ $13t^2 + 13t - 78 = 0$ $\Rightarrow$ $t^2 + t - 6 = 0$ $\Rightarrow$ $t = -3, 2$

16_. 

$\left(1 \pm 2\cos\frac{\pi}{6}, \sqrt{3} \pm 2\sin\frac{\pi}{6}\right)$

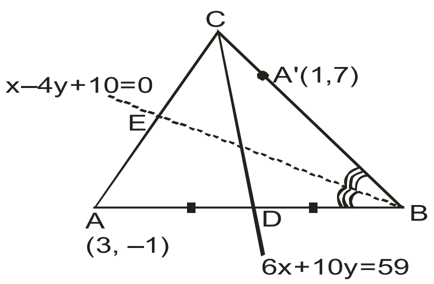
17_. 

Image of A(3, –1) about line BE is

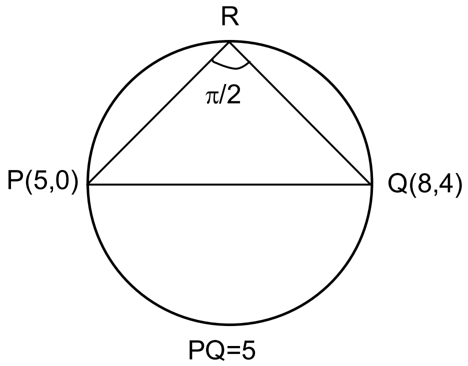
$\frac{x-3}{1} = \frac{y+1}{-4}$ = –2(1) $\Rightarrow$ (x, y) $\equiv$ (1, 7) lies on side BC.

Let vertex B is $(4\alpha - 10, \alpha)$.

Mid point of AB lies on 6x + 10y = 59 $\therefore$ $\alpha = 5$ $\Rightarrow$ B(10, 5)

18_. Equation of C_1 : $x^2 + y^2 - 2ax - 2ay + a^2 = 0$. Equation of C_2 : $x^2 + y^2 - 2bx - 2by + b^2 = 0$

19_.



Maximum area of ΔPQR = $\frac{1}{2} \times 5 \times \frac{5}{2}$ = 6.25 sq. units

20_. Let equation of circle is $x^2 + y^2 + \lambda(x - y) = 0$. Radius = $\sqrt{\frac{\lambda^2}{4} + \frac{\lambda^2}{4}} = 1$ $\Rightarrow$ $\lambda = \pm \sqrt{2}$

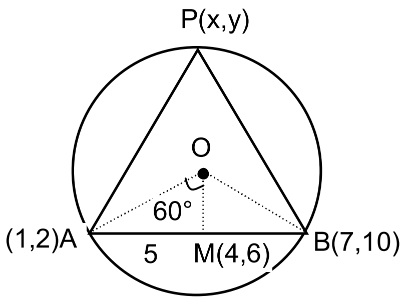
21_. Let $x - 2 = \cos\theta$ & $y - 2 = \sin\theta$

22_*. Equation of median through A is $y - 4 = -2(x - 1)$ $\Rightarrow$ $y = -2x + 6$

Let coordinates of point A is $(\alpha, 6 - 2\alpha)$

Now, $\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ \alpha & -1 & 3 \\ 6-2\alpha & 3 & 5 \end{vmatrix} 1 = 5$ $\Rightarrow$ $\alpha = 0, 2$

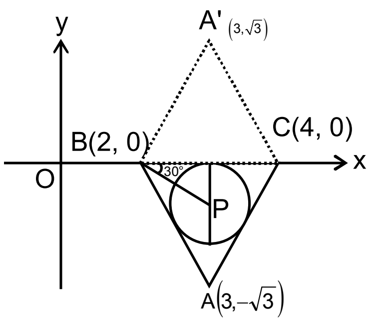
23_*.



OA = 5cosec 60° = $\frac{10}{\sqrt{3}}$ OM = 5cot 60° = $\frac{5}{\sqrt{3}}$

24^*. If z lies inside triangle then ℓ, m, n are all of same sign

25^*.



tan 30° = $\frac{PQ}{1}$ $\Rightarrow$ PQ = $\frac{1}{\sqrt{3}}$ $\Rightarrow$ r = $\frac{1}{\sqrt{3}}$

$\Rightarrow \quad P\left(3\ ,\ -\frac{1}{\sqrt{3}}\right) \quad \Rightarrow \quad (x-3)^2 + \left(y \pm \frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3}$

26^*. Equation of tangent at (1, 2) is $13x - 9y + 5 = 0$

Required equation is $(x-1)^2 + (y-2)^2 + \lambda(13x - 9y + 5) = 0$

$\Rightarrow \quad x^2 + y^2 - (2 - 13\lambda)x - (4 + 9\lambda)y + 5(1 + \lambda) = 0$

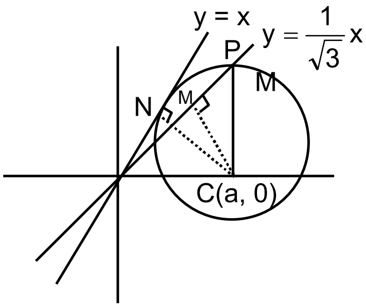
Radius = $\sqrt{\frac{(2-13\lambda)^2}{4} + \frac{(4+9\lambda)^2}{4} - 5(1+\lambda)} = \sqrt{\frac{5}{2}} \quad \Rightarrow \quad \lambda = \pm\frac{1}{5}$

equation of required circle is

$\Rightarrow \quad 5\{(x-1)^2 + (y-2)^2\} \pm 1(13x - 9y + 5) = 0 \quad \Rightarrow \quad 5x^2 + 5y^2 + 3x - 29y + 30 = 0$

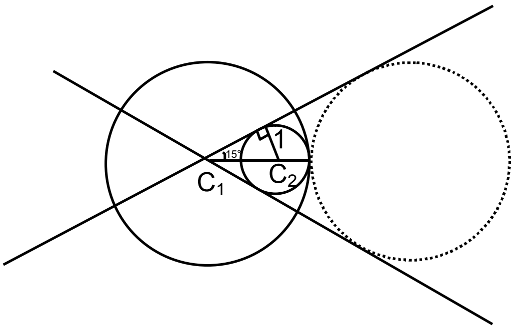
& $\quad 5x^2 + 5y^2 - 23x - 11y + 20 = 0$

27^*.



Radius = CN = $\frac{|a|}{\sqrt{2}} \quad \Rightarrow \quad CP^2 = CM^2 + MP^2 \quad \Rightarrow \quad \frac{a^2}{2} = \left(\frac{0 - \frac{a}{\sqrt{3}}}{\sqrt{1 + \frac{1}{3}}}\right)^2 + 1^2 \quad \Rightarrow \quad a = 2$

28^*.



$\text{cosec}15° = \frac{C_1C_2}{1} \quad \Rightarrow \quad r \pm 1 = \text{cosec}15° \quad \Rightarrow \quad r = \sqrt{6} + \sqrt{2} \pm 1$

29^*. Equation of tangent is $y = mx \pm 2\sqrt{1+m^2}$

$\Rightarrow \quad (\beta - m\alpha)^2 = 4 + 4m^2 \quad \Rightarrow \quad (\alpha^2 - 4)m^2 - 2\alpha\beta\, m + (\beta^2 - 4) = 0$

$\Rightarrow \quad m_1 + 2m_1 = \frac{2\alpha\beta}{\alpha^2 - 4}$ and $2m_1^2 = \frac{\beta^2 - 4}{\alpha^2 - 4}$

$\Rightarrow \quad 2\left(\frac{2\alpha\beta}{3(\alpha^2-4)}\right)^2 = \frac{\beta^2 - 4}{\alpha^2 - 4} \quad \Rightarrow \quad \frac{8\alpha^2\beta^2}{9(\alpha^2-4)} = \beta^2 - 4$

$\Rightarrow \quad 8\alpha^2\beta^2 = 9(\alpha^2\beta^2 - 4\alpha^2 - 4\beta^2 + 16) \quad \Rightarrow \quad \alpha^2\beta^2 - 36(\alpha^2 + \beta^2) + 144 = 0$

Disc of f(x) is $144 + \alpha^2\beta^2 = 36(\alpha^2 + \beta^2) > 0$ so (A) is not true

Locus of (α^2, β^2) is $xy - 36x - 36y + 144 = 0$ which is a hyperbola

As $\beta^2 = \frac{36(\alpha^2 - 4)}{(\alpha^2 - 36)} \quad \Rightarrow \quad \frac{\alpha^2 - 4}{\alpha^2 - 36} > 0 \quad \Rightarrow \quad \alpha \in (-\infty, -6) \cup (-2, 2) \cup (6, \infty)$

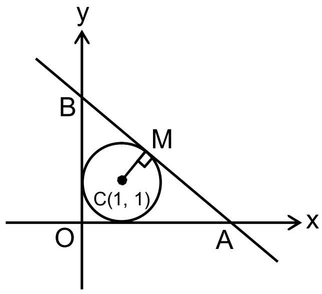
30^*. Let $y = mx$ be the chord.

Points of intersection of chord and circle are given by $(1 + m^2)x^2 - (3 + 4m)x - 4 = 0$

$\Rightarrow \quad x_1 + x_2 = \dfrac{3+4m}{1+m^2}$ and $x_1x_2 = \dfrac{-4}{1+m^2}$

As $\quad x_2 = -4x_1 \Rightarrow \quad 9 + 9m^2 = 9 + 16m^2 + 24m \quad \Rightarrow \quad 7m^2 + 24m = 0 \Rightarrow \quad m = 0, -\dfrac{24}{7}$

31.



Equation of AB is $\dfrac{x}{a} + \dfrac{y}{b} = 1 \quad CM = 1$

$$\Rightarrow \quad \left| \frac{\frac{1}{a} + \frac{1}{b} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right| = 1 \quad \Rightarrow \quad -\left(\frac{1}{a} + \frac{1}{b} - 1\right) = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$$

32^*. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ m-1 & m^2-7 & 5 \\ m-2 & 2m-5 & 0 \end{vmatrix} = m^3 - 4m^2 + 5m - 6 = (m-3)(m^2 - m + 2) \Rightarrow \Delta = 0 \quad \Rightarrow \quad m = 3$

33*. Area $= \left| \dfrac{1}{2} \begin{vmatrix} \frac{a}{m_1} & a & 1 \\ \frac{a}{m_2} & a & 1 \\ 0 & 0 & 1 \end{vmatrix} \right| = \left| \dfrac{a^2(m_1 - m_2)}{2m_1m_2} \right| = \left| \dfrac{a^2(a+2)}{2(a+1)} \right|$

34*. $3m^2 + am + 2 = 0$ and $6m^2 - bm - 4 = 0$

$\Rightarrow \quad m_1 + m_2 = \dfrac{-a}{3}, \; m_1m_2 = \dfrac{2}{3}, \; m_1 - \dfrac{1}{m_2} = \dfrac{b}{6}, \; \dfrac{-m_1}{m_2} = \dfrac{-2}{3}$

$\Rightarrow \quad (m_1m_2)\left(\dfrac{m_1}{m_2}\right) = \dfrac{4}{9} \quad \Rightarrow \quad m_1^2 = \dfrac{4}{9} \quad \Rightarrow \quad m_1 = \pm \dfrac{2}{3}$

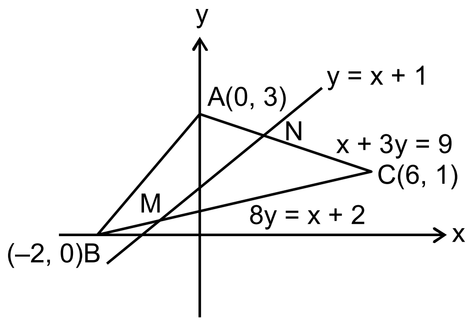
(i) $\quad m_1 = \dfrac{2}{3} \quad \Rightarrow \quad m_2 = 1 \quad \Rightarrow \quad a = -5, b = -2$

(ii) $\quad m_1 = -\dfrac{2}{3} \quad \Rightarrow \quad m_2 = -1 \quad \Rightarrow \quad a = 5, b = 2$

35*. As diagonals are bisectors of angle A so their equations are $\dfrac{x-y+k}{\sqrt{2}} = \pm\dfrac{7x-y+k}{5\sqrt{2}}$

As they pass through (1, 2) $\quad \Rightarrow \quad k = 0, 5/2$

36*.

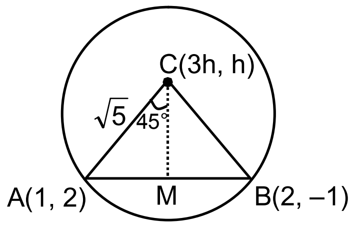


$N\left(\frac{3}{2},\frac{5}{2}\right)$ and $M\left(\frac{-6}{7},\frac{1}{7}\right)$ $\Rightarrow$ $\frac{-6}{7} < \alpha < \frac{3}{2}$

37*. Line meets x-axis at $A\left(-\frac{c}{a},0\right)$ & y-axis at $\left(0,\frac{-c}{b}\right)$

$\therefore$ $\frac{-c}{a} > 0$ and/or $\frac{-c}{b} > 0$

38*.



$AM = \frac{AB}{2} = \frac{\sqrt{10}}{2}$ $\Rightarrow$ $AC = \sqrt{5}$

CM equation of CM is x = 3y

let C(3h, h) $\Rightarrow$ $(3h-1)^2 + (h-2)^2 = 5$ $\Rightarrow$ h = 0, 1

Sol. **(39 & 40)**

Equation of line joining points A(3, 7) and B(6, 5) is $2x + 3y - 27 = 0$

Equation of family of circles S is $(x-3)(x-6) + (y-5)(y-7) + \lambda(2x + 3y - 27) = 0$

$\Rightarrow$ $x^2 + y^2 - 9x - 12y + 53 + \lambda(2x + 3y - 27) = 0.$

Equation of common chord $-5x - 6y + 56 + \lambda(2x + 3y - 27) = 0.$